Flexing on Topology, or, Contrapposto Architecture

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In 1977, mathematician Robert Connelly discovered a unique eighteen-sided closed, hinged polyhedron that, remarkably, wiggled, overturning centuries of mathematical discourse linking polyhedra with rigidity. This new category of polyhedra is the source of an ongoing interdisciplinary collaboration between architecture and discrete geometry. The flexible polyhedron serves as a generative design tool to develop a new approach to structure and a new relationship of the body in space, as well as an analytical lens through which to understand and challenge the history of architecture's close association with upright rigidity. In the late 1960s, the Architecture Principe Group (Paul Virilio and Claude Parent), argued against propriety and restraint with a theory of the body that challenged upright posture. The oblique function replaced a system of rigid proportions of a universal body at rest in Euclidean space with the figure of the dancer put in motion by the forces of gravity on the oblique. However, Virilio laments their non-orthogonal system of continuous folded planes became "all blobs, blobs, blobs."

Taking topological thinking filtered through Deleuze, Greg Lynn and others harnessed emerging computer technology to produce infinite variations of doubly-curved, amorphous spheres. The seamless smoothness of "animate form", however, ensured mathematical rigidity, a result of which Lynn is undoubtedly aware as he employs new robotic technologies to re-animate his forms. In this context, flexible polyhedra are both incredibly simple geometric forms in a world of complex-curved topological spheres; and much more complex, capable of flexibility without abandoning geometry or rigidity and without cutting-edge technologies. To borrow a phrase, the flexible polyhedron is both square and groovy. This ongoing interdisciplinary project reveals the false distinction between geometry and topology as interpreted by the architectural discipline, and explores the architectural ramifications of this new world of flexible polyhedra while using material/spatial practice to further understand and represent topology.

FLEX

In 1977, Robert Connelly stunned the mathematics community when he discovered a polyhedron that flexed. In particular, the polyhedron was made of 18 triangular faces attached along hinged edges. Until then, most mathematicians believed that all polyhedra were structurally rigid, influenced by the 1813 proof by Augustin Cauchy guaranteeing rigidity for the convex case. While the near entirety of polyhedra stand upright and rigid, the Connelly polyhedron, and its later simplification by Klaus Steffen, moved and slouched, and, depending on its orientation, appeared to fall to the side. Like the purposeful counterpoise popularized by the likes of Steve McQueen, this new family of flexible polyhedra adopt a contrapposto stance that similarly transgresses the norm of rigid, upright posture.

The architectural implications are significant and wide-ranging. The discipline of architecture from Vitruvius to Le Corbusier is largely, however implicitly, built on the presumption of rigidity, reinforced by the association of the body and the building and their shared investment in uprightness against the force of gravity.[1] It is so fundamental as to be rarely considered, let alone challenged.

A generation of architects emerging in the 1990s, led primarily by Greg Lynn, tackled and sought to displace uprightness and rigidity with fluid, folded surfaces. Influenced by topological thinking filtered through the writing of Gilles Deleuze, theorists of the fold set up a dichotomy between rigid, tectonic geometry and flexible, continuous topology exemplified by the form of the blob.[2] It is not without irony, however, that the curved animate forms generated in the computer, all but guaranteed geometric rigidity when they were baked and made tangible. In fact, the current interest in moving architecture by many of the same architects reflects the desire to recapture the fluid transformations achieved virtually twenty-five years ago.

Flexible polyhedra, although discovered before the architectural interest in the fold, reveal the false distinction between the rigidity of geometry and the flexibility of topology. They exhibit two extreme positions of rigidity with a range of motion that transforms the space it encloses. Using flexible polyhedra, we theorize a new relationship between the body and architecture that builds on Parent and Virilio's *oblique function*, one where both the body and the architecture are

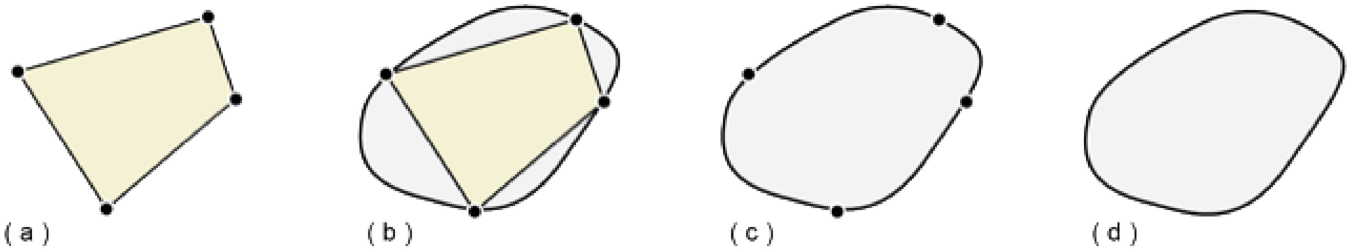

Figure 1. Diagrams moving from polyhedra to topological sphere. Image by authors.

put into motion and oscillate between moments of equilibrium and disequilibrium.[3] This work seeks to recuperate the interrogate and open architecture to new postures and bodies. Lastly, it opens up new possibilities for moving architecture that is both structurally rigid and flexible simultaneously without the use of external or internal apparatus.

"FOLDING IN ARCHITECTURE"

Topology was created and set in distinction to geometry most famously by Leonhard Euler with the "Königsberg bridge" problem. Euler dismissed typical geometrical data such as length and distance and instead examined only notions of adjacency to determine if it was possible to cross each of the seven bridges of Königsberg exactly once. Indeed, geometric data presented an obstacle to the bridge solution by introducing extraneous information, clouding the underlying structure.[4] In general, topology emphasizes the foundational structures such as adjacency, connectedness, orientability, and boundary, while geometry builds upon this with the added notion of length (denoted as a metric in mathematics), from which all means of measurements arise (such as area, volume, and angles). In his seminal work in the 19th century, Bernhard Riemann showed that a topological space allows for numerous types of metrics, each of which yields a different geometric space. For example, classical shapes such as spheres and cubes are both geometric in nature, with measurements of length allowing calculations of shortest distances between two points. Yet both shapes are identical topologically, and easily allow twisting, contortions, and the like while maintaining their identity as spheres.

Topologically defined surfaces and shapes served as the basis for an emerging architectural discourse in the 1990s framed in distinction to the extant dominant discourse of deconstructivism. Lynn argued: "Both Venturi and Wigley argue for the deployment of discontinuous, fragmented, heterogenous and diagonal formal strategies based on incongruities, juxtapositions and oppositions within specific sites and programmes."[5] A new dichotomy was constructed that put divergent figures like VSB and Wigley along with Tschumi and Gehry under an umbrella of tectonic geometry in contradistinction to the new continuous topological surface capable of absorbing complexity and difference without dissolving into a homogeneous form. Lynn continues: "Pliancy allows architecture to become involved in complexity through flexibility. It may be possible to neither repress complex relations of differences with fixed points of resolution nor arrest them in contradictions, but sustain them through flexible, unpredicted, local connections."[6] The baking technique of folding was used as a metaphor for the way ingredients can be woven together while remaining separate. Distinct, measurable geometric shapes (Fig. 1a) could be draped and connected with a continuous surface that folds over and absorbs them without losing the underlying geometry (Fig. 1b).

A series of roof structures designed by Shoei Yoh served as an example of the way a continuous surface could integrate particularities like structural spans, beam depths, lighting, lateral loading, and ceiling heights. Rather than average or unify the disparate elements, they were absorbed into a continuous roof surface. Yoh's roof structures were conceptually compelling examples of the promise of a topological approach to architecture, a smooth surface that could incorporate deformations and different spatial/formal/structural conditions as necessary, producing a unique but arbitrary curved geometry.[7]

The theory of the topological fold intersected with emerging computer technology, specifically the incorporation of differential calculus algorithms in computer aided design software to produce the blob. The blob came to exemplify the formal/spatial realization of the theory of the fold, a natural extension from Yoh's roof structures that saw the removal of any evidence of geometry in favor of an amorphous sphere subjected to force parameters (Fig. 1d). Blobs became a new category of forms "defined not by what they [were], but by the way they change[d] and by the laws that describe[d] their constant variations."[8]

Despite endless geometric variation, however, the blob is always, topologically speaking, the same. In fact, it is quite primitive from a topological standpoint. Nevertheless, the notion of a surface that dissolved traditional distinctions like floor, wall, roof, was newly enabled by the computer. Further, in the case of the *Embryological House*, an early example of

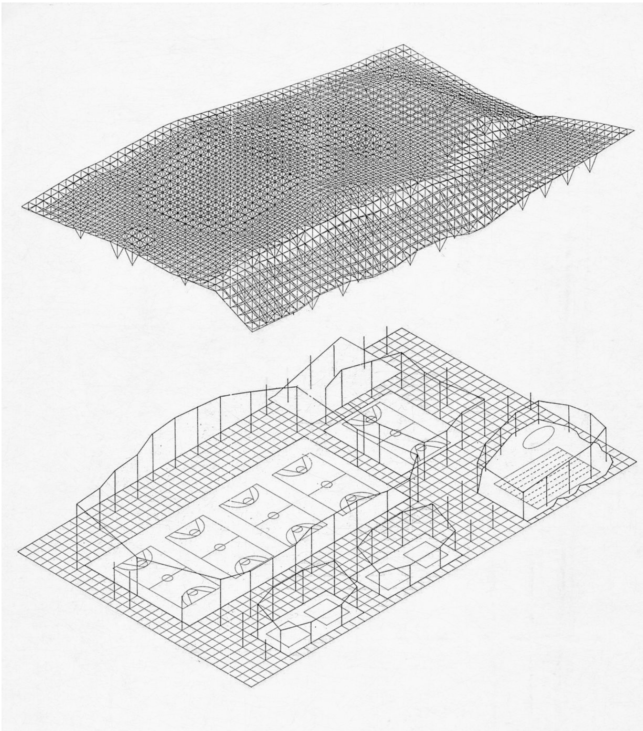

Figure 2. Shoei Yoh. *Exploded Axonometric, Odawara Municipal Sports Complex.* 1990-1991.

blobitecture, we see how disinterest in the geometry of the form enabled a new approach to variation and customization. Bernard Cache, paralleling Lynn, explains that “there ceases to be a static plan or model from which objects are made. Instead there emerges a ‘nonstandard mode of production’ where changing parameters in the computer make possible ‘a different shape for each object in the series’.”[9] The topological blob surface is not only smooth in spatial terms, but its deformation is continuous temporally.

Unfortunately, these explorations were truncated and focused primarily on the perception of motion that harkened back to a moment of actual animation in the computer. As Carpo explains, “it is about creating built forms, necessarily motionless, which can nevertheless induce the perception of motion by suggesting the ‘continual variation’ and ‘perpetual development’ of a ‘form becoming’.”[10] It is not difficult to understand why all flexibility and motion ceased when form exited the computer into physical space, but the emphasis on complex curvature foreclosed the possibility of topological thinking to open up a new line of inquiry to challenge and unlearn the disciplined, militarized, able, upright body.

INTERDISCIPLINARITY

The established discourse on folding became the source for a new collaboration for us, a architect/architectural historian and a geometer/topologist mathematician. A couple years ago, we began to meet and engage in a series of conversations around interesting intersections between mathematics and architecture. Topics ranged across the board, but the concept of folding surfaces resonated between us as it constituted an already robust historical example of topological thinking in architectural theory and design. Casual conversations became more formalized when we co-taught a course in the Spring of 2019. The title of the seminar-studio course, *Folding*, reflected this initial conceptual connection but also the ways in which we sought to expand and complicate it. Topics paired mathematical theorems with architectural histories, covering the relationship between linkages and space frames, uniqueness and interiority, and of course, rigidity and the fold.

However, as we taught the class, we realized that our shared set of terms like rigidity, flexibility, smoothness, discreteness, and continuity, were often defined differently, or held so many definitions as to carry little meaning. In particular, the use of the notion of continuity, so fundamental to the theory of folding in architecture, produced significant confusion in the discipline of mathematics due to its rigorous formulation in calculus. Conversely, the mathematical notion of discreteness, foundational to issues of measurability and computation, appeared in architecture merely to be a semantic distinction.

Ultimately, the course served as a framework for a now ongoing collaborative book project, of which flexible polyhedra represent one chapter. For us it serves many functions: a means to unpack and clarify a set of terms that cut across architecture and math, a lens through which to reimagine the relationship between the body and space, and a challenge to develop a new approach to structure. The work is new, and ongoing, and we are excited to share it and welcome your feedback.

THE OBLIQUE

Architecture, as a discipline, is largely and historically built on an association with the upright, postured body. Vitruvius went so far as to claim that it was precisely that humans stood rigidly erect that enabled the development of architecture: “Many came together into one place, having from nature this boon beyond other animals, that they should walk, not with head down, but upright, and should look upon the magnificence of the world and of the stars.”[11] Since then, uprightness, as evidenced in the image of the Vitruvian Man, has haunted and influenced architecture and humans just the same. The outstretched human set within a cube illustrates the link between upright posture and structural rigidity. He remains poised within a similarly rigid cube even as you tip, tumble, or apply force.

That’s not to say that the body has always been, or continues to be, restricted by upright posture. Starting in the mid-19th century, upright posture was tackled and dismantled, culminating with the association of the relaxed body with consumerism in the mid-20th century.[12] Architecture, too, has flirted with the prioritization of comfort over propriety, from the addition

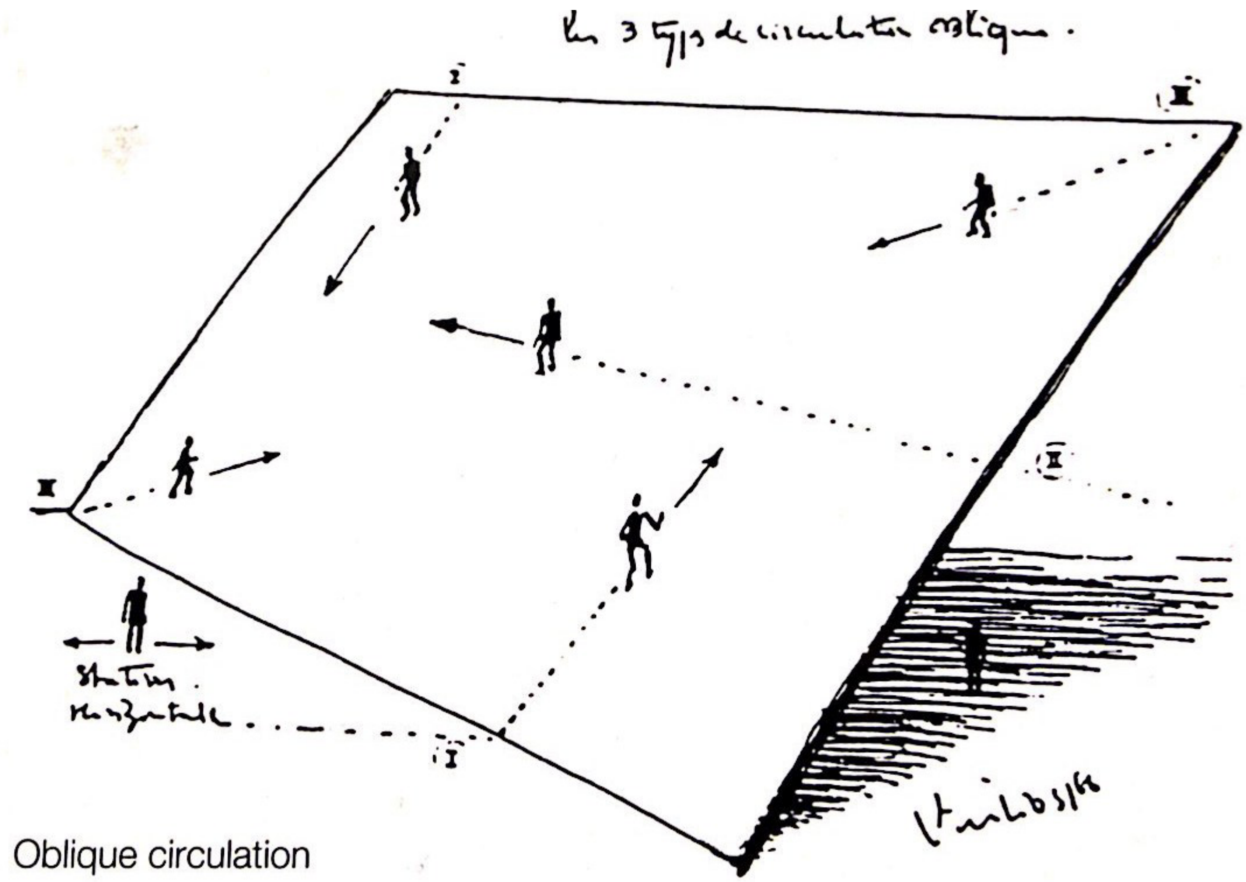

Figure 3. Architectural Principe Group. *Oblique Circulation*. 1971.

of upholstering and springs in furniture to the discourse of inflatable architecture that encouraged humans to recline if not simply lay down and sprawl out. It is not coincidental that the lasting image of Archigram's *Suitaloon* featured a relaxed and reclined David Greene, a human engaging in relaxed consumption from a relaxed posture. Juxtaposed to the Vitruvian Man, the flexible bubble surrounding a reclined body registers a new alignment of architecture and the body invested in the collapse of upright rigidity. Nevertheless, the influence of the rigid upright body on architecture largely persists, most explicitly in the primacy of the right angle. For Le Corbusier, "the orthogonal and the rectilinear are geometric defenses against the random, material, and dirty effects of accidental relations… Anything that deviates is oblique…"[13] Hence the epitome of Modernist form, a cubic polyhedron.

A higher-dimensional version of a polygon, the polyhedron is a region of space bounded by a finite number of flat polygonal faces. The most famous polyhedra are the five Platonic solids, all of which are convex, where line-of-sight between two points is always maintained. In 1813, Cauchy proved a remarkable property about polyhedra: if a convex polyhedron is built by simply matching pairs of edges of its flat polygonal face plates, a geometrically unique polyhedron would appear.[14] In particular, although the angles along the edges are not provided by these matching instructions, they would automatically be determined. Thus, geometric information remarkably appears from simple matching information. His theorem asserts that any convex polyhedron constructed from face plates hinged along edges is rigid and unable to flex.

In the late 1960s, Paul Virilio partnered with Claude Parent to form the Architectural Principe Group (APG). Although it was short-lived, they produced a manifesto in which they detailed the *oblique function*.[15] In direct reaction to the reign of the right angle, APG theorized oblique, hinged planes as an alternative to orthogonality. And even while the oblique, as ramp, featured prominently in Modernist design, it was largely deployed as a distinct and highly regulated secondary architectural object that served to reinforce the static orthogonality of the primary architectural space.[16] Rejecting the two fundamental axes of Euclidean space, they proposed instead a single axis, the oblique, capable of achieving both horizontal and vertical conditions simultaneously. Applied to architecture and the built environment, the oblique blurred the floor with the wall and the wall with the roof, delivering not only an entirely new kind of space, but a form that was almost entirely traversable. No longer was the inhabitant of the oblique restricted to accessing certain planes like the floor, or made to exit one to enter another, as in the case of the ramp. In the architecture of the oblique, inhabitants seamlessly moved from plane to plane, each constantly redefined and reoriented as floor became wall became roof became floor, all the while retaining rigidity.

Cauchy may have settled the matter of rigidity for convex polyhedra, but over the next 150 years, mathematicians wondered about the nonconvex case. Every nonconvex example that was constructed proved to be rigid, though a general theory was not established. Herman Gluck proved in 1975 that "almost all" polyhedra were rigid, making it tantalizingly feasible that rigidity is independent of convexity.[17] But when Robert Connelly announced the construction of a flexible polyhedron just two years later, the mathematics community was stunned.[18] Modifying a self-intersecting flexible octahedron (designed in 1897 by the French engineer Raoul Bricard), Connelly found the first example of a true flexible polyhedron, consisting of 18 triangular faces.[19] Subsequent reductions led to a 14-triangle, 9-vertex flexible nonconvex polyhedron constructed by Klaus Steffen, the simplest example possible.[20]

The unique capability of flexible polyhedra forces architecture to confront its long-standing presumption and reliance on rigidity. Although Virilio and Parent don't explicitly frame the *oblique function* as a product of topological thinking, and only begin to articulate the new continuous fluidity, the pair undoubtedly produced a kind of topological surface not unlike those by later theorists of the fold. The oblique does not smoothly transition from plane to plane like the stereotypical image of the topological surface, yet Henry Cobb explains how a sharp crease can still be the product of folding a continuous surface: "A fundamental rule for an architectural manifestation of folding is that the folding must never occur at a joint between the elements which make up the surface to be folded. A joint is a void between two pieces and cannot be folded. Any fold which coincides with a joint is not a fold but the manipulation of two separate pieces."[21] Although this is but a semantic distinction topologically, it does reveal how the oblique might be understood topologically from within the architectural discourse, despite the presence of sharp, defined edges. We might add an additional diagram to *Figure 1* that illustrates a flat-sided polyhedra without the designated decomposable points at the corners. In fact, material continuity was fundamental to the *oblique function*, with examples often registered in poured concrete or tectonic structures wrapped with carpet.

The oblique even entailed a shift in design from geometric proportions to gravitational force. Contrary to architecture associated with figures like the Vitruvian Man or "le modulor" that produced a system of rigid mathematical proportions of a universal body at rest, APG associated the oblique with the figure of Neitzsche's dancer.[22] By tilting the ground plane, APG theorized, the force of gravity was made felt. The body was no longer allowed to remain easily rigid and upright; movement is induced and only made more or less difficult depending on the body's orientation to the plane. The oblique, a challenge to upright form, challenged the upright body, throwing it into disequilibrium. Diagrams were supplemented and brought to life by Nicole Parent who developed a style of dance on the oblique illustrating how instability could yield new potential bodily movements and new moments of equilibrium between the body and the space.[23] In the still moment between moving up and down the oblique, Nicole Parent takes on a contrapposto stance. With hips thrown to one side, the opposing leg that would normally bend at the knee, now fully extends to meet the sloping plane. Herein, the body achieves equilibrium, like the Vitruvian Man, but without rigid uprightness.

The contrapposto stance that once constituted a rebellious public presentation for its looseness in a world of upright, rigid posture, becomes now a more natural and beneficial stance on the oblique. Using the body's linkages to slouch to the side achieves a counterpoise to the slope. The benefits of the new relationship of the body to the space proposed by APG extended even to daughter Chloe Parent. Even though the oblique was never quite realized at a large scale in a ground-up building, Claude Parent infilled the living areas of his family's apartment. A long-time sufferer of vertigo, she recalls today how growing up on the oblique did not have the intended effects of disequilibrium. In fact, quite the opposite, it aided her balance.[24] Although we don't do disability studies, we recognize and are interested in the potentialities of the oblique, and further, the flexible polyhedra, for further intersectional work that challenges the primacy of the upright, disciplined, abled body in architecture.

The *oblique function*, unfortunately, was rather short-lived. Despite proposing a certain kind of continuity, playing with mass and force, and theorizing a new relationship between the body and space, it would be later reframed as merely a precursor to the Deleuzian fold and blobitecture. Virilio accepts and laments the legacy of their topological approach: "Today the oblique is everywhere and it's a catastrophe. But only because of what has been made of it - it's all blobs, blobs, blobs."[25] We offer up flexible polyhedra as an alternative legacy to the *oblique function*. Flexible polyhedra, as architectural space, have the potential to extend the oblique and the contrapposto stance. The contrapposto body in motion is put in relation to a contrapposto form also in motion. Like blobs, flexible polyhedra are topologically the same: continuous, folded spheres. Subjected to force, the apparently rigid polyhedron flexes under the pressure. Oriented to produce a flat floor, the polyhedron gives way to the oblique. As it slouches into a contrapposto stance, so too does the body inside. Counterpoise in the form forces counterpoise in the body, and vice versa.

Moreover, unlike blobs, which eliminated geometry in favor of differential calculus to produce virtual flexibility, flexible polyhedra maintain measurable geometry while being also

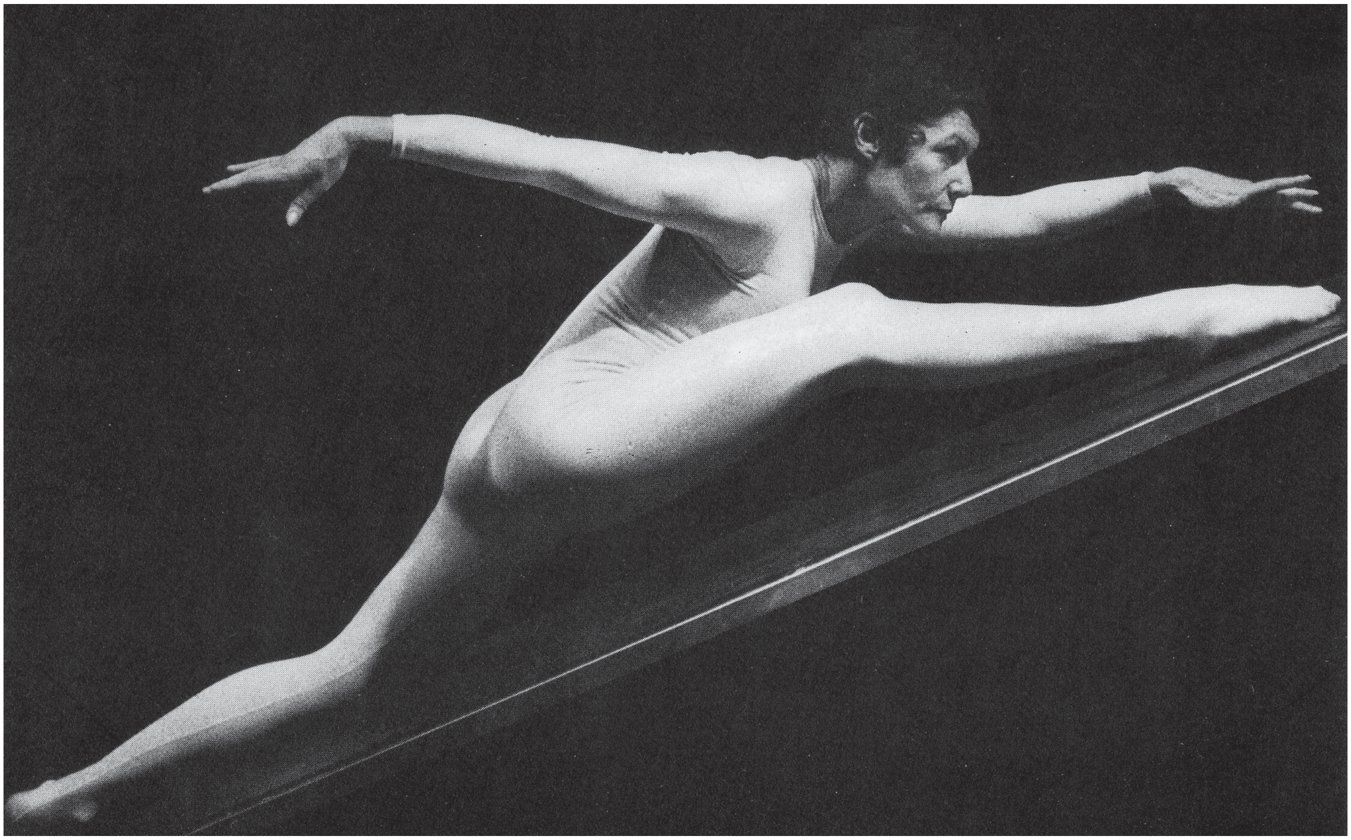

Figure 4. Nicole Parent. L'Inclipan. 1971.

physically flexible. Importantly, flexible polyhedra are distinct from collapsible structures in that they are capable of transforming the space inside without changing the amount. In 1997, Idzhad Sabitov proved that polyhedral volume does not change as it flexes as we might assume.[26] In flexible polyhedra, Jeffrey Kipnis gets his more inclusive architecture: "I'm tired of every building telling me I should be young and fit and have good posture. I'm not young, I'm not fit, I like sitting hunched over, I'm often drunk and I like to lean on stuff."[27] Flexible polyhedra actively slouch alongside their inhabitants.

FLEXIBLE POLYHEDRA

Returning to the blob, flexible polyhedra reveal the false distinction in the dichotomy set up between geometry/tectonics and topology/force. Put simply, and to borrow a phrase, flexible polyhedra are both square and groovy.[28] Continuous, non-developable surfaces aligned with emerging digital tools to produce "animate forms" generated through a manipulation of forces rather than measured geometries, even if the resultant structure retained, or regained rigidity. Forms are today rolled, stretched, bent, and more, in attempts to reanimate them to their pre-baked state in the computer. The same thinking and design approach that unleashed a radical collection of double-curved surfaces put them back into the world of geometry, just with new complex contributions.[29]

Architecture remains fundamentally a geometric product. This is not necessarily a surprise, nor a problem. But, the same blobs that were so good at simulating movement now require amazing amounts of technology to animate them because of their double-curved geometric complexity. It is not without irony that virtual flexibility foreclosed the possibility of actual flexibility. In the past quarter-century, with the advent and growth of computational powers, there has been a movement away from the continuous (blobs) to the discrete (polyhedra). The underlying reason is the emphasis on discretization, the process of making continuous features of an object into discrete features, making them suitable for computations. In particular, a continuous object (Fig. 1d) goes through two types of transformations: First, a decomposition into finite pieces, for data storage (Fig. 1c). Yoh's roof structures actually take on new importance by maintaining the geometric "creases" that allow for decomposability of the surface while allowing for the recreation of the smooth folded surface; Second, a linearization of these pieces, for ease of computation (Fig. 1a). Most calculations and computations in geometry are handled in the linear category (points, lines, planes, polygons), moving from geometric analysis (such as calculus) into algorithmic analysis.

Thus, the movement of roof covering (of polyhedral structures) to topological blobs is now fully reversed, with a harkening back to flat structures. In the midst of this, the flexible polyhedron

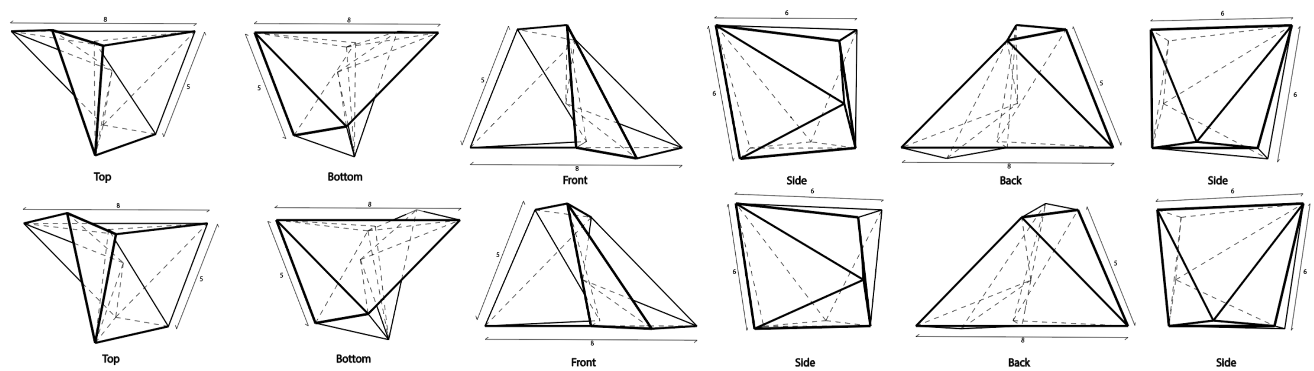

Figure 5. Elevations of Steffen Polyhedra. Image by Makenzie Nickel.

offers a way forward for the oblique that leads to literal movement that is at the same time both simpler geometrically than doubly-curved blobs, and more complex because it is continuous both in space and time. Some of the greatest jewels of mathematics have come from recovering geometric data from an object's discretized state. The works by Euler in the 1750s (classification of manifolds), by Carl Friedrich Gauss in the 1820s (curvature of surfaces), and by Marston Morse in the 1920s (critical point analysis) show that discrete structures are worthy of pursuit.[30]

In 2013, *Log* published a conversation between Lynn and Peter Eisenman, in which he claimed: "If I can take a ride in a driverless car on a public street, then I see no reason my building can't wiggle a little."[31] His *RV House* from the same year is essentially the animation of a previously baked animate form. A rigid spherical shell is spun and rotated on a dual-axis gyro base. And yet, the house abides by and reinforces the notion of upright rigid posture. Rather than tilt the plane and force the body into non-upright positions and motion, the form seamlessly rotates under the occupant's feet with the body remaining perfectly upright in space. By contrast, the flexible polyhedron yields a kind of dumb animation. It requires no elaborate robots nor complicated tectonics, being at the same time both rigid (along its faces) and flexible (along its edges).[32] Further, flexible polyhedra propose a new relationship between the body and space. It does not move, or shift, except under the exertion of force from the body. That is to say, it enters into a contrapposto relationship to the body. In between the extreme ends of flexibility where rigidity of the object reveals itself, a weighted balance is required. Where in the *oblique function* the architecture is rigid and the body is put into motion, and in the *RV House* the architecture is put into motion and the body remains upright, the flexible polyhedron forces the space and the body into a contrapposto stance together.

The research presented herein represents just the beginning and is meant to unpack, clarify, and theorize the flexible polyhedron as it intersects with the discipline of architecture. Like Deleuze's fold, we see significant potential for new kinds of architectural form, and more importantly, a new kind of relationship between the body and space. Our research is ongoing as we incorporate undergraduate research students to explore the structural and spatial possibilities. We are currently working on the design and fabrication of a full-scale temporary pavilion to test materials, connections, and the effects of transformable space on the body. The process parallels a similar, unbuilt proposal by APG to build and temporarily inhabit an oblique form to test the theory of the *oblique function*.

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